Summary of Univalent Relational Gauge Dynamics: A Finite Groupoid Field Theory Deriving Gauge Symmetry, Defect Matter, and Emergent Geometry from Coherent Identity Transport

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In recent decades, category theory and homotopy type theory have supplied rigorous languages for describing structure, identity, and transformation, yet converting these languages into explicit, computable physical models has proven difficult. This paper constructs Univalent Relational Gauge Dynamics as a minimal finite framework that inverts the usual explanatory order of theoretical physics. Configurations are defined on a finite oriented 2-complex by assigning local identity objects to vertices and invertible transport morphisms to oriented edges. Gauge transformations appear as local automorphisms that merely relabel presentations without altering physical content. Curvature is realized exactly as the failure of transported identity to return to itself after traversing an elementary face. Stable nontrivial holonomy sectors constitute defect matter, and the least presentation-independent coherence cost required to sustain such a sector supplies a candidate definition of mass. A conjugation-invariant action governs the dynamics and reduces precisely to ordinary lattice gauge theory with Wilson plaquette action in the single-object groupoid limit. Quantization is expressed as an automorphism-weighted sum over distinct physical equivalence classes rather than over redundant presentations. The model is deliberately finite and exactly computable, separating established mathematical structures from new physical hypotheses while providing concrete numerical tests, gauge-invariance checks, scaling studies, and explicit falsification criteria. By treating admissible equivalence as the primary dynamical ingredient, the framework offers a controlled arena in which gauge structure, particle-like excitations, mass gaps, and relational geometry may emerge from one underlying law of coherent identity transport.

Keywords: Univalent Relational Gauge Dynamics, groupoid field theory, identity transport, coherence action, curvature as failed identity closure, defect matter, mass as minimum coherence cost, lattice gauge theory recovery, finite quantization, emergent geometry, homotopy type theory, equivalence-first ontology, topological defects, relational distance, thermodynamic gravity conjecture.

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1. The Equivalence-First Paradigm Shift in Fundamental Physics

The development of modern theoretical physics has been marked by successive enlargements of the notion of symmetry and equivalence. From the Galilean and Lorentz transformations of classical and relativistic mechanics, through the internal symmetries of quantum field theory, to the diffeomorphism invariance of general relativity, the principle that physically distinct configurations may be related by transformations that do not alter observable predictions has proven extraordinarily fruitful. Yet in nearly all existing frameworks this principle is introduced after the basic ontological ingredients have already been chosen: a spacetime manifold is posited, fields are defined upon it, a gauge group is selected, and only then is a quotient by the action of that group imposed to identify equivalent descriptions. The present work explores the consequences of reversing this order. Equivalence relations, or more precisely the structured collections of admissible identifications that constitute physical identity, are taken as primary. Local presentations of physical structure appear as particular realizations within a larger web of equivalences, and the dynamics of the theory governs the formation, transport, and obstruction of those equivalences rather than the evolution of fields on a pre-existing background. This reversal carries both technical and conceptual consequences. Technically, it suggests that the configuration space of the theory should be a groupoid or homotopy quotient from the outset rather than a set that is later quotiented. Conceptually, it raises the possibility that gauge symmetry, particle content, mass, and even geometric structure might emerge as stable features of a single dynamics of coherent identification instead of being inserted as separate postulates.

1.1 Historical Context and Conceptual Motivation

The idea that physical identity might be constituted by equivalence rather than by intrinsic properties has deep roots in both the philosophy of physics and in mathematical developments of the twentieth century. In gauge theories the observation that the vector potential is defined only up to the addition of a gradient led to the recognition that the true degrees of freedom reside in the holonomies around closed loops. Lattice formulations made this structure discrete and computationally accessible, demonstrating that confinement and other non-perturbative phenomena could be studied by summing over equivalence classes of link variables. Parallel developments in topological quantum field theory showed that entire physical theories could be formulated in purely categorical terms, with observables arising from morphisms in suitable categories rather than from pointwise field values. More recently, homotopy type theory has supplied a formal language in which identity itself is represented by paths, and higher identities by higher paths, while the univalence axiom identifies equivalent structures within the foundational system. These threads suggest that a fully relational foundation for physics, in which the admissible transformations between local descriptions are the fundamental data,

may be attainable. The motivation for pursuing such a foundation is both practical and conceptual. On the practical side, the persistent difficulties in quantizing gravity, in understanding the origin of the Standard Model gauge group and fermion content, and in giving a precise account of measurement and probability within quantum theory all point to the possibility that the conventional starting point, in which a background geometry and a fixed symmetry group are assumed, may be too rigid. On the conceptual side, the success of category-theoretic methods in organizing diverse physical theories raises the question whether a single, uniform categorical dynamics can generate the structures that have hitherto been introduced separately. If admissible equivalence is treated as the primary dynamical ingredient, then gauge freedom appears as the automorphism structure of realized identity types, curvature appears as the obstruction to coherent transport, and stable obstructions appear as matter. Each of these phenomena is thereby placed on the same ontological footing rather than being added piecemeal.

1.2 Limitations of Imposed Symmetry in Standard Gauge and Field Theories

Despite their empirical success, conventional formulations of gauge theory and general relativity share a common architectural feature that limits their explanatory power. A spacetime or lattice is chosen first; a principal bundle or a set of link variables is introduced; a specific structure group is selected; and gauge invariance is then enforced by hand, either through the introduction of Fadeev-Popov ghosts, through restriction to gauge-invariant observables, or through explicit summation over orbits. The particular gauge group that appears in nature, the number of fermion generations, the pattern of symmetry breaking, and the existence of a mass gap are all inserted as initial data rather than derived from a more primitive dynamical principle. Moreover, the continuum limit is taken after the symmetry has been imposed, so that questions about whether gauge symmetry itself can emerge from a discrete or finite underlying structure are difficult to pose within the formalism. In quantum gravity approaches that begin with a fixed differentiable manifold, the problem of background independence is addressed by imposing diffeomorphism invariance after the fact, yet the very notion of a smooth manifold already encodes a substantial amount of geometric structure that one might hope to recover rather than presuppose. These limitations are not merely aesthetic. They manifest in the inability of current theories to predict the values of fundamental constants, to explain the hierarchy of scales, or to provide a unified account of matter and geometry. A framework in which symmetry and geometry arise as consequences of a dynamics of equivalence would, if successful, address these explanatory gaps directly. In addition, the conventional separation between the choice of gauge group and the dynamics of the fields makes it difficult to ask whether a given symmetry is dynamically preferred or merely consistent. When multiple identity types with different automorphism groups are permitted from the outset, competition among sectors becomes a dynamical question whose answer can be read from free-energy comparisons or from the

stability of defect sectors. The conventional architecture, by fixing the group in advance, removes this question from the domain of dynamics altogether.

1.3 Objectives and Standards of Success for a Constructive Model

The central objective of the present paper is therefore to construct a single, minimal, finite model in which the following phenomena are derived rather than postulated: the appearance of local gauge symmetry from the automorphisms of local identity objects; the interpretation of curvature as the failure of identity transport to close coherently; the identification of stable topological obstructions with particle-like matter; the definition of mass as the minimum invariant cost of sustaining such obstructions; and the formulation of quantization as summation over physical equivalence classes with appropriate automorphism weighting. In addition, the model is required to recover ordinary lattice gauge theory in a well-defined limit, to be exactly computable on finite complexes, and to yield at least one nontrivial result that is not inserted by definition, such as a preferred gauge sector, a quantized defect spectrum, or a nonzero mass gap that survives refinement. Four layers must be kept distinct throughout: the established mathematics of groupoids, lattices, and homotopy theory; the interpretive claim that these structures describe physical identity and equivalence; the dynamical hypotheses encoded in the coherence action; and the conjectural links to continuum physics and gravity that remain to be established. Success is measured by explicit, reproducible criteria. The action and all physical observables must be invariant under local changes of presentation. The one-object reduction must reproduce the Wilson plaquette action up to normalization. At least one complete finite partition function must be calculable by direct enumeration or summation. And the proposed physical quantities, above all the mass defined by minimum coherence cost, must be independent of arbitrary choices of representative, triangulation, or labeling. These standards are deliberately stringent, for only by meeting them can the equivalence-first approach be shown to constitute a genuine alternative rather than a re-description of existing theory. The finite setting is chosen precisely because it permits every symbol in the model to receive a concrete computational meaning and because it allows direct verification that gauge invariance is not merely asserted but enforced by the form of the action itself. Once these internal standards are satisfied on small complexes, controlled scaling studies can be used to investigate whether the emergent structures persist or diverge in the large-complex limit, thereby providing a bridge between the exact finite theory and the continuum conjectures that motivate the broader program.

2. The Finite Oriented 2-Complex as Relational Computational Scaffold

The equivalence-first ontology requires a concrete arena in which admissible identifications can be assigned, composed, tested for coherence, and varied under local automorphisms. The present framework supplies this arena in the form of a finite oriented 2-complex. This complex functions as a combinatorial scaffold rather than as a discretization of a pre-existing spacetime manifold. Its vertices record distinct local contexts or presentations of identity; its oriented edges record the possibility of coherent transport between neighboring contexts; and its faces record the minimal closed paths along which the consistency of transport can be checked. Because the complex is finite, every assignment of identity objects and transports can be enumerated exactly, every gauge orbit can be counted with its automorphism stabilizer, and the full partition function can be computed by direct summation. Because it is oriented, the direction of transport and the sense of circulation around faces are unambiguously defined, allowing the formation of composite paths and the extraction of holonomy without additional sign conventions. The scaffold therefore makes every ingredient of the subsequent theory—configurations, gauge transformations, curvature, defects, and the coherence action—mathematically explicit and computationally accessible from the outset. Geometry is not assumed at this stage; it is permitted to emerge later from the costs and densities of the transports that realize low-action configurations.

2.1 Definition and Combinatorial Data of the 2-Complex

A finite oriented 2-complex K consists of three finite collections together with structure maps that encode incidence and orientation. Let V be the set of vertices, E the set of oriented edges, and F the set of oriented faces. Each edge e in E is equipped with a source vertex s(e) in V and a target vertex t(e) in V. The reverse orientation of e is denoted e^{-1} and satisfies the reversal rules s(e^{-1}) = t(e) and t(e^{-1}) = s(e). Each face f in F is equipped with an ordered cyclic boundary partial f = (e_1, e_2, ..., e_n) consisting of oriented edges such that the target of each edge coincides with the source of the next: t(e_i) = s(e_{i+1}) for i = 1 to n-1, with the final edge returning to the initial vertex so that t(e_n) = s(e_1). The opposite orientation of the face is written f^{-1} and carries the reversed boundary (e_n^{-1}, ..., e_1^{-1}). These data may be collected into the single tuple K = (V, E, F, s, t, partial). No metric, length, or area is attached to the cells at this stage; such quantities, if needed, enter only as coefficients or weights inside the coherence action. The complex may be realized as a triangular mesh, a square lattice, an irregular cellulation, or any other finite decomposition whose incidence relations satisfy the above axioms. Its only essential property is that it supplies a finite, oriented, composable collection of paths against which the coherence of identity transport can be tested.

2.2 Role of Vertices, Edges, and Faces in Encoding Adjacency and Closure

Within the equivalence-first interpretation, the cells of the complex acquire direct physical meaning as loci of identity and transport. A vertex v supports a local identity object A_v that represents the type of structure realized at that context. Different vertices may carry different identity objects, allowing the model to accommodate heterogeneous phases or distinct automorphism sectors. An oriented edge e from v to w supports an invertible transport morphism U_e that identifies the structure at v with the structure at w in a reversible manner. Composition of such morphisms along concatenated edges yields the transport of identity along longer paths. Because the edge is oriented, the inverse transport is unambiguously assigned to the reversed edge, enforcing consistency under orientation reversal. A face f supplies the smallest closed path along which transport can be tested for coherence. The composite transport around the boundary of f produces a holonomy automorphism of the identity object at the base vertex. When this holonomy equals the identity automorphism, identity has been transported consistently around the elementary loop; when it differs from the identity, a local obstruction to coherence has been detected. Thus vertices encode local identity, edges encode admissible transport, and faces encode the elementary consistency conditions whose violation defines curvature. Global topological features of the complex, such as non-contractible cycles that cannot be filled by faces, encode obstructions that cannot be removed by local adjustments and therefore survive as stable defect sectors.

2.3 Boundary Conditions, Topology, and Their Physical Interpretation

The boundary subcomplex partial K of the finite 2-complex provides a natural locus for imposing external constraints. Fixed transport morphisms along boundary edges may represent an imposed external gauge field or a preparation context. Fixed identity objects on boundary vertices may select particular physical phases or asymptotic conditions. Nontrivial holonomy prescribed on the boundary can force the appearance of compensating defects in the interior. Periodic boundary identifications create toroidal or higher-genus topologies in which globally nontrivial transport sectors become possible even when every elementary face is flat. Open boundaries permit flux to enter or leave the system and therefore allow the study of defect creation and annihilation at the edge. Mixed boundary conditions can be used to model preparation and measurement contexts within a single finite complex. Because the complex is finite, every choice of boundary data yields a well-defined finite-dimensional space of configurations whose physical equivalence classes can be enumerated exactly. Global topology enters the dynamics through the existence of cycles that cannot be contracted through the faces; transport around such cycles may remain nontrivial even when every face holonomy is trivial. The model therefore distinguishes local curvature, measured by face holonomies, from

global topological sectors, measured by holonomies around non-bounding cycles. Both types of obstruction contribute to the coherence action and to the classification of stable defects.

2.4 Why Finiteness Enables Exact Computation and Controlled Scaling

Finiteness is adopted not as an approximation but as a methodological advantage that makes every claim of the theory subject to direct verification. On a finite complex the complete set of configurations is finite whenever the identity groupoid is finite, or can be discretized in a controlled manner when the groupoid is compact. Gauge orbits can be counted exactly by Burnside enumeration or by explicit orbit-stabilizer calculations, and the automorphism-weighted partition function can be evaluated by summation rather than by Monte Carlo sampling. Gauge invariance of the action and of all observables can be checked by applying randomly generated local automorphisms and confirming numerical invariance to machine precision. Defect sectors can be classified by exhaustive search over holonomy conjugacy classes and by computation of their minimum coherence costs. Scaling studies are performed by systematically enlarging the complex, refining its triangulation, or increasing its genus while holding boundary conditions fixed; the behavior of the mass gap, defect density, correlation length, and effective dimension can then be tracked as functions of system size. Because the underlying definitions remain exact at every scale, any divergence or convergence observed in these quantities is a genuine property of the model rather than an artifact of an uncontrolled approximation. The finite setting therefore supplies both a rigorous testing ground for the internal consistency of the equivalence-first hypotheses and a controlled laboratory in which the conjectural emergence of continuum geometry and gravity can be investigated step by step.

3. Local Identity Groupoids and Admissible Equivalences

The finite oriented 2-complex supplies the combinatorial skeleton on which local contexts are adjacent and on which closed paths can be tested for coherence. The algebraic structure that organizes the admissible identifications between these contexts is a groupoid of local identity types. A groupoid consists of a collection of objects together with morphisms between them that are invertible and composable whenever the target of one morphism coincides with the source of the next. In the present framework the objects represent possible local identity types realized at the vertices of the complex, while the morphisms represent reversible physical identifications that can be transported along the edges. Because every morphism possesses an inverse, the structure automatically encodes the reversibility of physical transport. Because morphisms compose along paths, the transport of identity along composite routes is unambiguously defined. The groupoid therefore provides a uniform language in which local presentations, their mutual identifications, and the automorphisms that leave a given

presentation invariant are treated on the same footing. When the groupoid contains only a single object, its morphisms reduce to the elements of an ordinary group and the framework recovers conventional gauge theory. When the groupoid contains several objects with distinct automorphism groups, the possibility arises that an effective gauge symmetry is selected dynamically rather than imposed at the outset.

3.1 Objects as Local Identity Types and Morphisms as Reversible Identifications

Each vertex v of the 2-complex is assigned a local identity object A_v drawn from the collection of objects of the groupoid G. This object encodes the type of structure that is being compared and transported at that location; it is not itself a field value or a particle state but the formal carrier of all admissible self-identifications and comparisons with neighboring locations. To each oriented edge e from vertex s(e) to vertex t(e) is assigned an invertible morphism U_e belonging to the hom-set Hom_G(A_s(e), A_t(e)). The morphism U_e records a coherent identification between the identity type at the source and the identity type at the target. Because the morphism is invertible, the reversed edge e^{-1} carries the inverse morphism U_e^{-1} from A_t(e) back to A_s(e). Composition of morphisms is defined whenever the target of one coincides with the source of the next: if e_2 follows e_1 along a path, the composite transport is the morphism U_e2 composed with U_e1. This composition is associative and unital, with the identity morphism at each object serving as the unit. The resulting structure is therefore a groupoid whose morphisms are precisely the admissible transports of local identity along the edges of the complex. Different choices of which morphisms are declared admissible correspond to different physical assumptions about which identifications are realizable.

3.2 Automorphism Groups and Their Interpretation as Local Gauge Freedom

For any object A in the groupoid, the set of morphisms from A to itself forms the automorphism group Aut(A). These automorphisms represent the reversible self-identifications of a given local identity type; they are the transformations that leave the local structure intact while possibly altering its presentation. In the model a local gauge transformation at vertex v consists of the choice of an element g_v belonging to Aut(A_v). Under such a choice the transport morphism along an outgoing edge e transforms according to the rule U_e maps to g_t(e) composed with U_e composed with the inverse of g_s(e). The same rule applied around a closed face shows that the face holonomy transforms by conjugation: the new holonomy is g_v composed with the old holonomy composed with the inverse of g_v. Consequently any function of the holonomy that is invariant under conjugation, such as the trace in a representation or the conjugacy class itself, remains unchanged. The automorphism group at each object therefore plays exactly the role of a local gauge group, yet it arises internally as the self-equivalence structure of the identity type realized at that vertex rather than being selected externally. When all vertices carry the same object A, every automorphism group coincides with a single group G equal to

Aut(A) and the model reduces to ordinary lattice gauge theory with structure group G. When distinct vertices carry distinct objects, the local gauge groups may differ and transitions between them become possible dynamical processes.

3.3 Heterogeneous versus Homogeneous Groupoids and the Possibility of Dynamical Selection

A homogeneous groupoid contains only one object; every local identity type is of the same kind and every local automorphism group is isomorphic to the same abstract group G. In this case the entire framework specializes to a conventional G-valued transport theory on the 2-complex. A heterogeneous groupoid, by contrast, contains several distinct objects A_1 through A_n, each equipped with its own automorphism group Aut(A_i) and its own space of admissible morphisms to the other objects. The configuration of the theory must now specify not only the transport morphisms but also which identity object is realized at each vertex. The coherence action can be extended by a term that assigns a local cost to the choice of object at each vertex. When this term is included, the relative stability of different sectors can be compared by evaluating the total action or the associated free energy. A sector in which a particular object A_i and its automorphism group dominate may be dynamically selected if configurations realizing that object systematically achieve lower action values while remaining compatible with the imposed boundary conditions and topology. Such selection constitutes a groupoid-level mechanism for spontaneous gauge symmetry breaking or for the emergence of an effective gauge group without that group having been inserted as the unique starting choice. The heterogeneous setting therefore enlarges the explanatory scope of the model: the specific gauge symmetry observed in nature can in principle be recovered as the stable outcome of a competition among admissible identity types rather than as an initial postulate.

3.4 Admissibility Selectors and the Distinction Between Formal and Physical Equivalence

Not every morphism present in the abstract groupoid G need correspond to a physically realizable identification. The model therefore introduces an admissibility selector that extracts, for each pair of objects A and B, a subset E_phys(A, B) of the full hom-set Hom_G(A, B). The selector is required to be compatible with the groupoid operations: it must contain all identity morphisms, be closed under composition whenever both factors are admissible, and contain the inverse of every admissible morphism. In addition it must preserve the observables and boundary conditions that define the physical problem under consideration. The resulting admissible morphisms are the only ones permitted to appear as edge transports in a physical configuration. Two configurations that differ by a family of local automorphisms are regarded as physically equivalent precisely when they are related by admissible gauge transformations; configurations related only by non-admissible morphisms lie in distinct physical sectors. The admissibility selector therefore implements the physical content of the equivalence-first ontology: it encodes which identifications are allowed to participate in the dynamics of

coherent transport and which are excluded on grounds of topology, boundary data, or conservation laws. Different choices of selector correspond to different physical theories sharing the same underlying groupoid; the selector is part of the specification of the model rather than a derived consequence. Once the selector is fixed, the space of physical configurations becomes a well-defined groupoid in its own right, and the coherence action and quantization procedure are defined directly on the admissible sector.

4. Field Configurations, Transports, and Gauge Transformations

With the finite oriented 2-complex providing the combinatorial scaffold and the groupoid supplying the algebraic structure of admissible identifications, a field configuration is obtained by assigning concrete data from the groupoid to the cells of the complex. The resulting assignment encodes the local identity types realized at each context together with the coherent transports that relate neighboring contexts. Because the transports are morphisms in the groupoid, they are automatically invertible and composable along paths. Gauge freedom enters as the freedom to alter the presentation of each local identity type by an automorphism without changing the underlying physical situation. The space of all such assignments, quotiented by the action of local automorphisms, constitutes the physical configuration space of the theory. This construction ensures that every notion introduced later—holonomy, curvature, defects, and the coherence action—refers directly to gauge-invariant features of the assigned data rather than to arbitrary choices of labeling.

4.1 Assignment of Identity Objects and Invertible Edge Transports

A configuration on the 2-complex K consists of two primary assignments drawn from the identity groupoid G. To each vertex v is assigned an object A_v of G, representing the local identity type realized at that context. To each oriented edge e with source s(e) and target t(e) is assigned an invertible morphism U_e belonging to the admissible hom-set E_phys(A_s(e), A_t(e)). The morphism U_e records how the identity structure at the source is coherently identified with the identity structure at the target. Optional local matter data ψ_v may be included at each vertex, taking values in a representation space associated with the object A_v; such data transform covariantly under the automorphisms of A_v. The pair consisting of the assignment of objects A_v and the assignment of admissible morphisms U_e fully specifies the configuration. Because every morphism in the groupoid is invertible by definition, the transport along any oriented edge can be reversed without additional structure. The configuration therefore encodes both the local types of identity and the network of reversible identifications that relate them across the complex.

4.2 Orientation Reversal and Consistency Requirements

Orientation reversal must be handled consistently for the configuration to be well-defined. For every oriented edge e the reverse edge e^{-1} carries the inverse morphism: the transport assigned to e^{-1} is required to be exactly the groupoid inverse of the transport assigned to e. This rule ensures that transporting identity from v to w and then immediately back from w to v recovers the original identity at v up to the identity automorphism. Along any closed path the composite transport is obtained by composing the morphisms in the order of traversal; the consistency requirement guarantees that the composite around a path traversed in one direction is the groupoid inverse of the composite around the same path traversed in the opposite direction. These reversal and composition rules are inherited directly from the groupoid axioms and impose no additional dynamical constraints. They do, however, guarantee that the notion of holonomy around a closed face is unambiguously defined once a base vertex and an orientation are chosen. Violations of reversal consistency would render the notion of curvature ill-defined, so the configuration space is restricted to assignments that satisfy the reversal rule at every edge.

4.3 Local Changes of Presentation and the Transformation Law for Transports

A local change of presentation at vertex v consists of the choice of an automorphism g_v belonging to the automorphism group Aut(A_v) of the identity object assigned to that vertex. Such an automorphism represents a reversible relabeling of the local identity type that leaves all intrinsic properties of the type unchanged. Under a family of such choices, one at each vertex, the transport morphism along an edge e transforms according to the rule U_e maps to the composite g_t(e) composed with U_e composed with the inverse of g_s(e). This transformation law is the finite, groupoid version of the familiar gauge transformation rule for connection forms or link variables. It ensures that the physical identification recorded by U_e is preserved while the particular presentations at the source and target vertices are altered. Because the transformation is implemented by groupoid morphisms, it automatically respects composition along paths: the transformed composite transport along a path equals the composite of the transformed individual transports. Consequently, the holonomy around any closed face transforms by conjugation under the local automorphism at the base vertex. Any quantity built from holonomies that is invariant under conjugation, such as the conjugacy class itself or the value of a class function, remains unchanged by the local change of presentation.

4.4 Gauge Orbits and the Homotopy Quotient Structure of Physical Configuration Space

Two configurations are regarded as physically equivalent when one can be obtained from the other by a family of local automorphisms, one at each vertex. The set of all configurations related in this way forms a gauge orbit. The physical configuration space is therefore not the set

of all possible assignments but the quotient of that set by the action of the local automorphism groups. Because the automorphisms at different vertices may fail to commute with one another when composed along paths, the quotient is more accurately described as a homotopy quotient or groupoid quotient rather than a naive set-theoretic quotient. In this quotient each physical situation is represented by an entire structured collection of presentations together with the admissible transformations connecting them. Observables are required to be constant on each orbit, or more precisely to descend to well-defined functions on the quotient. The coherence action is constructed so that its value is likewise constant on orbits, guaranteeing that the dynamics itself is gauge invariant by construction. The homotopy quotient structure also supplies the correct weighting for quantization: when the partition function is formed by summing over physical configurations, each orbit is counted once, with a factor that compensates for the volume of its automorphism stabilizer. This procedure prevents highly symmetric configurations from being overcounted merely because they admit many equivalent presentations.

5. Path Transport, Face Holonomy, and the Meaning of Curvature

Once a configuration has assigned local identity objects to vertices and admissible invertible transports to oriented edges, the notions of path transport and face holonomy follow directly by composition. These operations extract the net effect of moving identity along extended routes and around elementary loops. Because the transports are morphisms in the identity groupoid, composition is associative and respects the groupoid axioms. The resulting holonomy around a face is an automorphism of the local identity object at the base vertex. Gauge transformations act on this holonomy by conjugation, so that its conjugacy class is independent of the choice of local presentation. When the holonomy equals the identity automorphism, identity has been transported consistently around the loop; when it differs from the identity, a local obstruction to coherence is present. This obstruction supplies the precise, finite meaning of curvature within the model: curvature measures the failure of transported identity to close trivially after traversing an elementary face. The same construction extends to longer closed paths, distinguishing local curvature from global topological obstructions that cannot be removed by local adjustments.

5.1 Composite Transport along Paths and the Definition of Face Holonomy

A path p on the 2-complex is a finite sequence of consecutively oriented edges e_1, e_2, ..., e_n such that the target of each edge coincides with the source of the next. The composite transport along p is the morphism obtained by composing the individual edge transports in the order of traversal: U_p equals U_en composed with ... composed with U_e2 composed with

U_e1. Because each U_e is invertible, the composite along the reversed path is the groupoid inverse of U_p. When the path p is closed, returning to its initial vertex v, the composite U_p is necessarily an automorphism of the identity object A_v assigned to that vertex. For an oriented face f whose boundary is the closed path partial f = (e_1, e_2, ..., e_n) based at v, the face holonomy H_f is defined to be this composite automorphism: H_f equals U_en composed with ... composed with U_e1, an element of Aut(A_v). The holonomy therefore records the net self-identification obtained by transporting the local identity once around the elementary loop that bounds the face. Different choices of base vertex on the same face yield conjugate holonomies, so the essential information resides in the conjugacy class of H_f within Aut(A_v).

5.2 Conjugation Invariance of Holonomy and Presentation Independence

Under a local gauge transformation given by an automorphism g_v of A_v at the base vertex, the face holonomy transforms by conjugation: the new holonomy H_f' equals g_v composed with H_f composed with the inverse of g_v. This follows immediately from the transformation law for individual edge transports and the associativity of composition. Consequently, any function of the holonomy that is invariant under conjugation remains unchanged by the local change of presentation. Examples include the conjugacy class of H_f itself, the value of any class function such as the trace in a representation of Aut(A_v), and the order of H_f when the automorphism group is finite. Because these invariants do not depend on the particular choice of g_v at each vertex, they descend to well-defined quantities on the physical configuration space obtained by quotienting by gauge transformations. Presentation independence is therefore automatic for all observables and action terms constructed from conjugacy-invariant functions of face holonomies. This property is essential for the coherence action introduced in the next section, which must itself be gauge invariant by construction.

5.3 Curvature as the Obstruction to Trivial Identity Closure

The face is said to be flat when its holonomy H_f equals the identity automorphism id of A_v. In this case the composite transport around the boundary returns every local presentation exactly to itself; identity has closed coherently after one full circuit. When H_f differs from the identity automorphism, the transported identity fails to match its original presentation after traversing the face. This mismatch constitutes a local obstruction to coherence and is interpreted as curvature. The interpretation is precise and finite: curvature is the failure of identity transport to close trivially around an elementary loop. Nontrivial holonomy may be confined to individual faces or may extend over larger regions when neighboring faces share consistent but nontrivial holonomies. In either case the obstruction is detected by comparing the composite transport with the identity morphism at the base vertex. Because the holonomy transforms by conjugation, the presence or absence of curvature is independent of the choice of local presentation. The same notion applies to any closed path, not merely to the boundaries of

faces: nontrivial holonomy around a non-contractible cycle signals a global topological obstruction that cannot be removed by local gauge adjustments and therefore survives as a stable defect sector.

5.4 Higher Coherence Data and the Introduction of 2-Morphisms

When a surface bounded by a given closed path admits several distinct decompositions into elementary faces, the composite transports obtained from different decompositions may differ even though each individual face holonomy is well-defined. In such cases the model is extended by introducing 2-morphisms that relate the alternative composite transports. A 2-morphism is a morphism between morphisms in the groupoid; it compares two ways of transporting identity along the same overall path and records the obstruction or equivalence between them. Consistency conditions on these 2-morphisms around higher-dimensional cells generate higher curvature and higher obstruction data. The introduction of 2-morphisms is postponed until after the ordinary groupoid model has been solved, because the basic notions of configuration, gauge transformation, holonomy, and curvature are already fully defined at the level of 1-morphisms. When required, the higher data supply a refined notion of coherence in which not only the net transport but also the homotopy between different routes of transport is taken into account. This refinement connects the present construction to higher gauge theory and to the full apparatus of homotopy type theory, while preserving the computational accessibility of the finite 2-complex setting for the ordinary groupoid dynamics.

6. The Univalent Relational Coherence Action

The dynamics of the model is encoded in a single coherence action whose value on any configuration measures the total cost of maintaining admissible identity transport. The action is required to be invariant under local gauge transformations, local on the finite 2-complex, bounded from below in its Euclidean form, exactly computable, sensitive to nontrivial holonomy, and capable of supporting stable defect sectors when topology or boundary conditions prevent complete trivialization. These requirements are satisfied by constructing the action from three principal contributions: a face term that penalizes failures of identity closure around elementary loops, transport and matter terms that enforce covariant consistency of local data, and optional identity-selection, boundary, and defect terms that distinguish different topological sectors. The resulting action is conjugation invariant by construction and therefore descends to the physical configuration space obtained by quotienting by gauge transformations. Its minima and its statistical mechanics define the stable configurations and the partition function of the theory.

6.1 Design Requirements: Gauge Invariance, Locality, Boundedness, and Sensitivity to Nontrivial Holonomy

Any acceptable action must remain unchanged when a configuration is replaced by a gauge-equivalent one. Because gauge transformations act by conjugation on face holonomies and by the standard covariance rule on transported matter data, every term in the action is built from conjugation-invariant or covariantly transforming quantities. Locality requires that the action be a sum of contributions each of which depends only on data assigned to a single vertex, edge, or face; no nonlocal kernels or long-range interactions are introduced at this stage. Boundedness from below in the Euclidean signature guarantees that the partition function is well-defined and that Monte Carlo or exact enumeration methods converge. Exact computability follows from the finiteness of the complex and the groupoid: when both are finite the configuration space is finite or discretizable and the action takes only finitely many values. Sensitivity to nontrivial holonomy is achieved by assigning a strictly positive cost to any face whose holonomy differs from the identity automorphism; flat configurations can achieve zero face cost, while configurations containing stable defects pay a positive but finite minimum cost. These design requirements together ensure that the action implements the physical principle that realized structures favor the greatest admissible coherence compatible with topological and boundary constraints.

6.2 Face Coherence Term and Conjugacy-Invariant Cost Functions

The principal term that detects curvature is the face coherence contribution. For each oriented face f a nonnegative conjugacy-invariant function K is chosen on the automorphism group such that K applied to the identity automorphism equals zero. The face term is then defined as beta times the sum over all faces f of the face weight w_f multiplied by K of the face holonomy H_f. Here beta is a positive coupling constant that controls the energetic cost of failed identity closure, and the weights w_f may be chosen uniformly or according to geometric data such as face area when such data are later introduced. Because K is conjugacy invariant, the entire face term is unchanged under local gauge transformations. Common choices for K include the Wilson class function Re Tr(I minus rho of H_f) in a faithful representation rho, or the squared distance from the identity in a bi-invariant metric on the automorphism group when the group is compact. In all cases K vanishes exactly on the identity and is positive elsewhere, so that flat faces contribute zero while curved faces contribute positively. The face term therefore supplies a precise, gauge-invariant measure of total curvature whose minima are achieved precisely when every elementary face is flat.

6.3 Transport and Matter Terms with Covariant Differences

When local matter data psi_v are present at vertices, an additional term enforces that these data are transported consistently along the edges. For each oriented edge e a covariant difference is defined by D_e psi equals psi at t(e) minus rho of U_e applied to psi at s(e), where rho is the representation of the automorphism group associated with the matter field. The transport term is then kappa times the sum over all edges e of the squared norm of D_e psi. The positive constant kappa controls the stiffness of the matter field. A potential term sum over vertices v of V of psi_v may be added to favor particular values of the local matter data. Both the covariant difference and the potential are constructed so that they transform correctly under local gauge transformations, ensuring that the full matter contribution is gauge invariant. When no matter fields are present the transport and potential terms are simply omitted. The combination of face, transport, and potential terms already yields a rich dynamics in which curvature and matter interact through the shared transport morphisms.

6.4 Identity Selection, Boundary, and Defect Contributions

When the identity groupoid is heterogeneous, an identity-selection term sum over vertices v of M of A_v may be included, where M assigns a nonnegative cost to each possible identity object. This term biases the dynamics toward sectors realizing particular identity types and their associated automorphism groups. Boundary terms are added when fixed data are imposed on the boundary subcomplex; they may take the form of Lagrange multipliers that enforce prescribed holonomies or fixed identity objects on the boundary, or they may simply restrict the summation in the partition function to configurations satisfying the boundary conditions. Defect contributions distinguish configurations belonging to different topological sectors. They may be implemented either by adding a chemical-potential-like term that favors or disfavors nontrivial holonomy classes, or by performing the sum over configurations separately in each topological sector and comparing the resulting free energies. Together these optional terms allow the model to treat distinct physical phases, external fields, and stable defect sectors on an equal footing within a single action principle.

6.5 The Complete Finite Action and the Principle of Maximal Coherent Equivalence

Collecting all contributions yields the complete Euclidean action S_URD equals the face coherence term plus the transport and matter terms plus the identity-selection term plus boundary and defect terms. Explicitly, S_URD equals beta times sum over f of w_f times K of H_f, plus kappa times sum over e of the squared norm of D_e psi, plus sum over v of V of psi_v, plus sum over v of M of A_v, plus S_boundary plus S_defect. The action is finite, exactly computable on any finite complex and finite groupoid, and gauge invariant by construction. Its physical content is summarized by the principle of maximal coherent equivalence: low-action

configurations preserve the greatest admissible coherence compatible with boundary conditions, topology, and stable physical distinctions. The principle does not demand universal triviality. Nontrivial stable sectors may survive and even dominate when topology or boundary data prevent complete coherent closure; in such cases the minimum action within each sector defines the rest energy of the corresponding defect. The minima of S_URD and the weighted sum over its level sets therefore determine both the classical ground states and the quantum statistical mechanics of the model.

7. Special Case Recovery: Ordinary Lattice Gauge Theory

A necessary consistency condition for any proposed generalization of gauge theory is that it must recover the established theory in a well-defined limit. The present framework satisfies this condition by exhibiting ordinary lattice gauge theory as the special case in which the identity groupoid contains only a single object. In this reduction every local identity type is of the same kind, every transport morphism belongs to a single automorphism group, and the coherence action reproduces the Wilson plaquette action up to normalization and lattice weights. The recovery demonstrates that conventional gauge theory is contained inside the equivalence-first model rather than replaced by an unrelated formalism. At the same time the reduction makes explicit what is not yet derived: the particular gauge group realized in nature, the presence of chiral fermions, and the pattern of symmetry breaking all remain to be obtained from the more general heterogeneous dynamics.

7.1 One-Object Groupoid Reduction and Emergence of a Single Gauge Group

Consider the case in which the identity groupoid G contains only one object A. Every vertex is then assigned the same identity type A, and every edge transport U_e is necessarily a morphism from A to A. The set of all such morphisms coincides exactly with the automorphism group Aut(A), which may be denoted G. The model therefore reduces to a conventional G-valued lattice transport theory on the finite oriented 2-complex. Local gauge transformations at each vertex are elements g_v of this single group G, and the transformation law for transports simplifies to the standard rule U_e maps to g_t(e) composed with U_e composed with the inverse of g_s(e). Face holonomies likewise take values in G, and the entire configuration space is the familiar space of G-valued link variables modulo local gauge transformations. Because the groupoid now has only one object, there are no transitions between distinct identity types and no dynamical competition among different automorphism groups. The framework therefore specializes precisely to the setting of ordinary lattice gauge theory with structure group G.

7.2 Wilson-Type Face Action and Plaquette Formulation

Within the one-object reduction the face coherence term is specialized by choosing the cost function K to be a Wilson-type class function. For a unitary representation rho of G one defines K of H_f to be the real part of the trace of the identity minus rho of H_f. This choice is conjugacy invariant, vanishes when H_f is the identity, and is nonnegative for all group elements when rho is unitary. Substituting this K into the face term yields beta times the sum over faces f of w_f times Re Tr(I minus rho of H_f). Up to an overall additive constant and a rescaling of the coupling beta, this expression is exactly the Wilson plaquette action of conventional lattice gauge theory. The transport and matter terms, if present, likewise reduce to the standard covariant kinetic terms for matter fields in representation rho. Consequently the complete coherence action S_URD reduces to the standard lattice gauge theory action with Wilson plaquette term plus matter contributions. The reduction is exact on any finite complex and requires no additional limiting procedure.

7.3 Recovery Theorem and Its Interpretation

The recovery may be stated as a precise theorem. When the identity groupoid contains only one object A, every edge transport is an automorphism of that object, and the face cost function K is chosen to be the Wilson class function Re Tr(I minus rho of H_f) in a faithful unitary representation rho, the Univalent Relational coherence action reduces exactly to the action of ordinary G-valued lattice gauge theory with Wilson plaquette term. The proof follows by direct substitution: the general transformation law for transports collapses to the standard gauge transformation rule, the face term collapses to the plaquette action, and all gauge-invariance properties are preserved. The theorem establishes that conventional lattice gauge theory is recovered as a restricted phase of the equivalence-first model. In this phase there is only one identity type, only one automorphism group, a fixed lattice, and no dynamical transitions among object types. The recovery therefore shows that the more general framework contains established physics as a special case rather than contradicting it.

7.4 What Is and Is Not Derived in the Reduction

The recovery theorem demonstrates that any ordinary gauge theory with structure group G appears inside the equivalence-first structure once the identity groupoid is restricted to a single object and the cost function is chosen appropriately. It does not, however, derive why nature realizes one particular gauge group rather than another. The specific group G = SU(3) times SU(2) times U(1) of the Standard Model, the existence of three fermion generations, the pattern of electroweak symmetry breaking, and the values of the coupling constants all remain external inputs in the one-object reduction. These features can be addressed only by returning to the heterogeneous groupoid setting in which multiple identity types compete dynamically and in

which the selection of an effective gauge group becomes a dynamical outcome rather than an initial choice. The recovery theorem therefore serves two purposes: it confirms internal consistency with established lattice gauge theory, and it isolates the additional dynamical mechanisms, available only in the heterogeneous case, that must be invoked to explain the specific gauge structure observed in nature.

8. Stable Defects, Charge, and the Definition of Mass

In the equivalence-first framework, matter is not introduced as a separate field species but arises as stable obstructions to the trivialization of identity transport. When a configuration contains nontrivial holonomy that cannot be removed by admissible local gauge transformations while preserving boundary conditions and topology, the obstruction persists as a defect sector. These sectors are classified by topological invariants of the transport assignment. The response of a defect sector to local automorphisms defines its charge, while the least presentation-independent cost required to sustain the sector defines its mass. Because the cost is obtained by minimization over all equivalent presentations, mass becomes an invariant property of the defect class rather than an artifact of any particular labeling. The resulting mass gap between the vacuum and the lightest defect provides a finite, computable analogue of the mass gap problem in conventional gauge theories and supplies a concrete test of whether particle-like excitations can emerge from coherence dynamics alone.

8.1 Defects as Localized or Global Obstructions to Trivial Transport

A defect is defined as any localized or global obstruction to trivializing the transport assignment. The simplest example is a single face whose holonomy H_f differs from the identity automorphism. More generally, a defect may consist of a cluster of faces whose collective holonomies cannot be gauged away simultaneously, a non-contractible cycle carrying nontrivial holonomy even when every elementary face is flat, or a failure of consistent gluing across overlapping patches of the complex. In each case the obstruction is detected by the existence of a closed path whose composite transport lies outside the identity component of the automorphism group. Because the complex is finite, every such obstruction can be identified by exhaustive enumeration of holonomy values and their conjugacy classes. Defects are therefore not postulated but discovered as stable features of low-action configurations that cannot be deformed into the vacuum sector without violating admissibility or boundary conditions.

8.2 Topological Invariants and Classification of Defect Sectors

Defect sectors are classified by topological invariants of the transport assignment. These invariants include the conjugacy class of face holonomies, the winding numbers of transports around non-contractible cycles, cohomological obstructions in the group cohomology of the

automorphism group, and global gluing data that record whether local trivializations can be assembled into a globally consistent section. Because the invariants are unchanged under local gauge transformations, they label entire gauge orbits and therefore label distinct physical sectors. On a finite complex the complete set of defect sectors is finite and can be enumerated by computing the possible values of these invariants over all admissible configurations. Each sector may be further subdivided according to its boundary behavior when open boundaries are present. The classification supplies a discrete spectrum of defect types whose stability and interactions can be studied by comparing the minimum actions within each sector.

8.3 Charge as the Representation Response of a Defect under Local Automorphisms

Charge is defined as the response of a persistent defect sector to local self-equivalences of the identity type. Given a defect sector D, one associates a representation rho_D of the automorphism group Aut(A) on a vector space V_D that encodes how the defect transforms under local gauge transformations. The representation may be obtained by examining the action of Aut(A) on the space of admissible transports that realize the defect or on the space of observables localized near the defect. The charge of the defect is then the set of quantum numbers furnished by this representation: the highest weight, the dimension, or the eigenvalues of generators when Aut(A) is a Lie group. Because the representation is defined on the defect sector itself rather than on an external matter field, charge emerges directly from the groupoid structure and the stability of the obstruction. Different defect sectors may carry different representations, yielding a spectrum of charges that is determined dynamically rather than inserted by hand.

8.4 Mass as the Minimum Invariant Coherence Cost over Equivalent Presentations

Mass is defined as the minimum invariant coherence cost required to sustain a given defect class. For a static defect sector D one considers the restricted action S_static obtained by fixing the topological invariants that label D and by imposing the same physical boundary conditions. The rest energy of the defect is then m_D c squared equals the infimum of S_static over all presentations belonging to D. The infimum is taken after all admissible gauge transformations have been applied, so that the resulting mass is independent of the choice of representative within the gauge orbit. Minimization removes arbitrary descriptive complexity: presentations that employ unnecessarily complicated labelings or redundant intermediate transports are automatically discarded in favor of the least-cost realization. The definition therefore implements the physical idea that mass measures the resistance of a coherent transport pattern to being deformed away, expressed as an equivalence-invariant energy cost. On a finite complex the infimum is attained and can be computed by enumeration or by optimization within each defect sector.

8.5 Stability Criteria and the Finite-Model Mass Gap

A defect sector represents particle-like matter only when it satisfies four stability criteria. It must be localized, meaning its support can be enclosed within a finite subcomplex whose complement admits a trivialization. It must be persistent under admissible perturbations, so that small variations of the transport data within the sector do not destroy the topological invariants. It must be protected against continuous removal, meaning there is no continuous family of admissible configurations that deforms the defect into the vacuum without crossing an infinite action barrier. Finally, it must carry a finite positive minimum cost, so that m_D c squared is strictly greater than zero. When these criteria are met, the difference between the global minimum of the action (the vacuum) and the lowest nontrivial defect minimum defines the finite-model mass gap. The gap is a computable quantity on any given complex. Its persistence or disappearance under controlled refinement of the complex and increase of system size provides a direct numerical test of whether the model generates a stable hierarchy of massive excitations from coherence dynamics alone.

9. Quantization as a Sum over Physical Equivalence Classes

Quantization in the present framework is formulated directly on the space of physical equivalence classes rather than on the larger space of all formal assignments. Because two assignments that differ by a local gauge transformation describe the same physical situation, the partition function is constructed as a sum over gauge orbits, with each orbit weighted by the reciprocal of its automorphism stabilizer volume. This procedure implements a finite analogue of groupoid cardinality and automatically corrects for overcounting of highly symmetric configurations. The resulting Euclidean partition function is exactly computable on any finite complex and finite groupoid. Gauge-invariant observables are obtained as weighted averages over the same sum. Real-time amplitudes are defined by analytic continuation after the Euclidean theory has been understood. The construction supplies a precise realization of the idea that quantum interference arises from the coherent superposition of distinct admissible identifications, while classical measurement corresponds to an effective truncation that discards higher relational information.

9.1 Finite Euclidean Partition Function with Automorphism Weighting

For a finite identity groupoid and a finite oriented 2-complex the Euclidean partition function is defined by summing over physical equivalence classes. Let [Phi] denote a gauge orbit of configurations, that is, an equivalence class under the action of local automorphisms. The partition function is then Z equals the sum over all such orbits of exp of minus S_URD of Phi divided by hbar, divided by the order of the automorphism group Aut of Phi of a representative

configuration in the orbit. The factor 1 over |Aut(Phi)| implements the groupoid-cardinality correction: configurations with large stabilizer groups are counted with reduced weight so that they are not over-represented merely because they admit many equivalent presentations. When the groupoid and complex are finite the sum is a finite sum and can be evaluated by direct enumeration or by systematic orbit counting. The resulting Z is a function of the coupling constants beta, kappa and of the boundary conditions; it encodes the complete statistical mechanics of the model at finite temperature.

9.2 Groupoid Cardinality Correction and Avoidance of Overcounting

The automorphism weighting prevents a common source of error in gauge theories: the overcounting of configurations that possess large symmetry groups. In an ordinary path integral without weighting, a configuration invariant under a large subgroup of gauge transformations would appear with a multiplicity equal to the volume of its orbit, artificially inflating its contribution. By dividing each term by the order of the stabilizer, the present sum counts each physical situation exactly once, weighted only by its Boltzmann factor. The correction is the finite-groupoid analogue of the Fadeev-Popov determinant or of the division by the volume of the gauge group in continuum formulations. Because the stabilizer order is an integer that can be computed exactly for each orbit, the procedure introduces no approximation and preserves the exact computability of the partition function. The same weighting appears in the definition of expectation values, ensuring that all observables are likewise free of overcounting artifacts.

9.3 Gauge-Invariant Observables and Expectation Values

A gauge-invariant observable O is a function on configurations that takes the same value on all members of a given gauge orbit, or more generally descends to a well-defined function on the quotient. The expectation value of such an observable in the Euclidean theory is the weighted average O equals one over Z times the sum over orbits [Phi] of O of Phi times exp of minus S_URD of Phi divided by hbar, divided by |Aut(Phi)|. Because both the action and the observable are constant on orbits, the expression is independent of the choice of representative within each orbit. Typical observables include face holonomies in representations, Wilson loops along longer paths, defect densities obtained by counting sectors with nontrivial invariants, and correlation functions of local matter operators. All such quantities are exactly computable on finite complexes once the complete set of orbits and their automorphism orders has been determined.

9.4 Real-Time Amplitudes and Interference among Admissible Identifications

After the Euclidean theory is understood, real-time amplitudes are obtained by the formal replacement of the Boltzmann factor exp of minus S divided by hbar with the phase factor exp of i S divided by hbar. The resulting sum over orbits defines transition amplitudes between

boundary configurations. Distinct admissible transport structures that connect the same boundary data contribute separate complex amplitudes; the total amplitude is their coherent sum. This construction realizes the physical proposal that quantum interference may be interpreted as interference among coherent identifications: each admissible way of transporting identity from one boundary to another contributes its own phase, and the observed probability is obtained from the modulus squared of the total sum. Because the sum is over physical equivalence classes rather than over redundant presentations, the interference is free of gauge artifacts. The Euclidean theory serves as the necessary regulator and provides the analytic continuation path that defines the real-time contour.

9.5 Measurement as Truncation of Higher Relational Structure

Classical measurement is modeled as an effective truncation that discards higher relational information. Formally one constructs a map tau_0 from the configuration groupoid M to the set of its connected components pi_0 of M. The map sends each configuration to the equivalence class of outcomes that can be distinguished by the chosen measurement apparatus. Probabilistic information is recovered from the push-forward measure on pi_0, while coherence information encoded in the higher morphisms of the groupoid is lost. The truncation therefore corresponds to the passage from a full relational description, in which multiple identifications remain superposed, to a classical description in which only coarse-grained distinctions survive. The information loss is quantifiable: the entropy of the fiber over each outcome measures the number of distinct admissible identifications that are identified by the measurement. In this way the Born rule and the emergence of classical probabilities appear as consequences of the truncation map applied to the quantized equivalence-first dynamics.

10. Dynamical Selection of Effective Gauge Structure

The one-object reduction recovers ordinary lattice gauge theory, yet it leaves the choice of gauge group as an external input. The full equivalence-first framework addresses this limitation by permitting heterogeneous identity groupoids in which several distinct object types, each carrying its own automorphism group, coexist and compete. The relative stability of these sectors is determined dynamically by comparing their free energies or their zero-temperature action minima. When one sector systematically achieves lower action values while remaining compatible with boundary conditions and topology, its automorphism group emerges as the effective gauge symmetry without having been imposed as the unique starting choice. Transitions between object types or reductions of automorphism groups supply a groupoid-level realization of spontaneous symmetry breaking. The mechanism therefore converts the question

of why a particular gauge symmetry exists into a dynamical question whose answer can be read from the minima of the coherence action.

10.1 The Problem of Postulated versus Selected Gauge Groups

In conventional formulations a gauge group is chosen at the outset and the dynamics is required to respect it. While this procedure yields consistent theories, it provides no dynamical explanation for the particular group that appears in nature. The Standard Model gauge group, the pattern of fermion representations, and the scale of symmetry breaking are all inserted by hand rather than derived from a more primitive principle. The equivalence-first model isolates this explanatory gap by distinguishing two layers. In the homogeneous, one-object case the gauge group is fixed by the choice of the single identity object, and the recovery theorem shows that ordinary lattice gauge theory is reproduced exactly. In the heterogeneous case multiple identity objects with distinct automorphism groups are permitted from the beginning, and the dynamics itself can favor one effective gauge structure over others. The heterogeneous setting therefore supplies the additional structure needed to convert gauge-group selection into a dynamical outcome.

10.2 Heterogeneous Identity Groupoids and Sector Competition

A heterogeneous identity groupoid contains several distinct objects A_1 through A_n, each equipped with its own automorphism group Aut(A_i) and its own space of admissible morphisms. A configuration must now specify both the transport morphisms and the choice of which identity object is realized at each vertex. The coherence action is extended by an identity-selection term that assigns a local cost M(A_v) to the object realized at vertex v. Different sectors of configuration space are then labeled by the dominant identity type or by the pattern of identity types across the complex. Because the action is additive, the total action of a configuration belonging to a given sector receives contributions from the face, transport, and identity-selection terms evaluated within that sector. The sectors therefore compete through the value of the action: configurations realizing one identity type may achieve systematically lower action than those realizing another type while satisfying the same boundary conditions.

10.3 Free Energy and Zero-Temperature Action Minima as Selection Criteria

The relative stability of competing sectors is quantified by their free energies or by their zero-temperature action minima. At finite temperature the free energy of sector i is F_i equals minus k_B T times the logarithm of the partition function Z_i restricted to that sector. The sector with the lowest free energy dominates the thermal ensemble. At zero temperature the comparison reduces to the minimum value of the coherence action within each sector. A sector whose minimum action lies below that of all other sectors is selected as the stable vacuum; its automorphism group becomes the effective gauge symmetry of the low-energy theory. Because

the minima are obtained by explicit minimization over admissible configurations, the selection is a dynamical result rather than an input assumption. The same comparison can be performed after the introduction of defects: sectors that accommodate stable defects at lower additional cost may be favored when the boundary conditions or topology require nontrivial holonomy.

10.4 Spontaneous Gauge Selection and Groupoid-Level Symmetry Breaking

When one sector systematically achieves the global minimum of the action, its automorphism group emerges as the effective gauge symmetry without having been postulated as the unique structure group. This constitutes spontaneous gauge selection at the groupoid level. Transitions between sectors correspond to changes in the dominant identity type and therefore to changes in the local automorphism groups. Such transitions realize a form of symmetry breaking in which the unbroken symmetry of one sector is reduced or altered in another sector. Because the transition is driven by the minimization of a single coherence action, the pattern of breaking is determined by the relative costs of the identity-selection term, the curvature term, and the defect terms rather than by an external Higgs potential. The mechanism therefore supplies a purely relational account of how an effective gauge group and its pattern of breaking can arise from the dynamics of admissible equivalence.

11. Relational Distance, Effective Dimension, and Thermodynamic Gravity Conjectures

Geometry in the present framework is not postulated as a background manifold but is permitted to emerge from the costs and densities of admissible identity transport. Once the coherence action and its minima have been defined, a candidate notion of relational distance can be extracted from the least-cost paths connecting distinct vertices or identity contexts. Effective dimension may then be read from the scaling behavior of volumes, spectra, and diffusion processes defined with respect to this distance. At a still later stage, the entropy associated with the multiplicity of presentations realizing the same coarse physical meaning can be compared with thermodynamic relations across effective causal boundaries. These constructions remain conjectural beyond the gauge sector; they are offered as controlled directions for investigation once the finite model has been solved exactly on small complexes and its scaling properties have been characterized.

11.1 Candidate Distance from Infimum Transport Cost

A candidate distance between two vertices v and w, or more generally between two identity contexts, is defined by taking the infimum of a cost functional C evaluated on admissible transports along paths connecting them. Explicitly, d(v,w) equals the infimum over all admissible paths p from v to w of C of U_p, where U_p is the composite transport morphism along p and C is a nonnegative function on morphisms that vanishes on identity morphisms and grows with

deviation from the identity. When the cost functional is chosen to be additive along composition or to satisfy a triangle inequality at the level of individual morphisms, the resulting d satisfies the axioms of a metric or a directed metric. In the general case the infimum need only be taken over paths that realize low-action configurations, thereby incorporating dynamical information into the geometry. The distance is relational because it depends only on the admissible identifications that actually participate in low-cost transport rather than on any pre-existing coordinate chart or metric tensor.

11.2 Metric Conditions and Possible Generalized or Causal Geometries

The candidate distance may or may not satisfy the classical metric axioms. Positivity and separation hold whenever C vanishes only on identity morphisms and the complex is connected by admissible paths. Symmetry may fail if the cost of transport in one direction differs systematically from the cost in the reverse direction, yielding a directed or causal geometry. The triangle inequality may be violated when the cheapest route between two points is not realized by a direct path but by a detour through a lower-cost region; in such cases the geometry is non-metric but still well-defined and useful for scaling analysis. These possible failures do not indicate a defect in the construction; they indicate that the emergent geometry may be generalized, causal, or asymmetric rather than Riemannian. The model therefore accommodates a broader class of geometries while still providing a precise operational definition of distance in terms of transport cost.

11.3 Estimation of Effective Dimension from Scaling and Spectral Properties

Effective dimension can be estimated from several independent scaling diagnostics once a distance or cost-based neighborhood structure has been defined. Ball-volume growth measures how the number of vertices reachable within cost radius r scales with r; power-law growth with exponent d indicates effective dimension d. Spectral dimension is extracted from the return probability of a diffusion process whose transition rates are inversely related to transport costs; the short-time behavior of the return probability yields the spectral dimension. Diffusion return probability, connectivity scaling, and the growth of low-cost equivalence neighborhoods supply additional independent estimates. When all diagnostics converge to the same value on sufficiently large complexes, the model exhibits a well-defined effective dimension that may differ from the topological dimension of the underlying 2-complex. Discrepancies between diagnostics signal the presence of anomalous diffusion or fractal structure generated by the coherence dynamics.

11.4 Entropy of Equivalence Fibers and Thermodynamic Relations

For any coarse physical meaning m realized by a set of presentations, the equivalence fiber is the preimage under the quotient map that sends each configuration to its physical equivalence

class. The entropy of this fiber is defined as S_eq of m equals k_B times the logarithm of the volume of the fiber, counted with automorphism-aware multiplicity. This entropy quantifies the number of distinct admissible identifications that realize the same coarse observable content. When an effective causal boundary is identified within the complex, the variation of S_eq across that boundary can be compared with thermodynamic relations of the form delta Q equals T d S_eq. If such a relation holds with a suitable identification of heat and temperature, the model supplies a thermodynamic origin for gravitational dynamics. The relation remains a conjecture at present; it is formulated only after the gauge sector has been solved exactly and after controlled scaling studies have established the existence of stable effective boundaries and well-defined fiber entropies.

11.5 Status of the Gravity Bridge and Required Further Derivations

The gauge recovery achieved in the one-object limit constitutes the first demonstrated result of the framework. Emergent geometry and thermodynamic gravity remain second-stage conjectures whose detailed implementation requires additional derivations. These include an explicit continuum limit in which the relational distance converges to a Lorentzian metric, a derivation of the Einstein equation or its thermodynamic equivalent from the variation of fiber entropy, and a demonstration that the resulting geometry is stable under refinement of the complex. Until these steps are completed, the gravity bridge is offered as a precisely formulated research direction rather than as an established consequence. The finite model nevertheless supplies the necessary controlled setting in which each required derivation can be tested numerically before any continuum extrapolation is attempted.

12. Implementation, Testing, Falsifiability, and Outlook toward a Theory of Everything

The equivalence-first model is constructed so that every definition, every invariance property, and every physical claim can be subjected to direct numerical verification on finite complexes. Implementation therefore begins with an explicit computational platform that supports the full data structures of the theory: vertex identity types, automorphism groups, oriented transport morphisms, face boundaries, holonomy computation, gauge transformations, defect classification, and exact counting of gauge orbits with automorphism weighting. Once implemented, the platform is used to run controlled baseline experiments, to measure primary observables, to verify gauge invariance numerically, and to perform scaling studies. The criteria for minimal and strong success are stated explicitly, as are the conditions that would require substantial revision or rejection of the proposed physical interpretations. The research sequence that follows from these tests leads from the finite kernel through higher structures,

continuum limits, and eventual phenomenology, while remaining clear about what has and has not yet been derived.

12.1 Computational Platform Requirements and Baseline Experiments

A computational platform must provide native support for the algebraic and combinatorial structures of the model. Vertices carry identity objects drawn from a finite or discretizable groupoid; each object is equipped with its automorphism group and representation theory. Oriented edges carry admissible morphisms together with the data needed for composition and inversion. Faces carry ordered boundary cycles so that holonomy can be computed by successive composition. Gauge transformations are generated locally at each vertex and applied uniformly across the complex. Defect classification requires routines that compute conjugacy classes, winding numbers, and cohomological invariants, together with orbit-stabilizer algorithms that count physical equivalence classes exactly. Baseline experiments include random transport assignments as null models, flat-coherence configurations in which every face holonomy is forced to the identity, single-object reductions that recover ordinary lattice gauge theory, heterogeneous groupoid runs that allow sector competition, and defect-seeded versus defect-free ensembles. Each baseline is run with multiple random seeds and complete parameter logging so that statistical significance and reproducibility can be assessed.

12.2 Primary Observables, Gauge-Invariance Tests, and Scaling Analyses

Primary observables are extracted directly from the enumerated orbits and their action values. They include the total coherence action, the distribution of face holonomies, the size of gauge orbits, the automorphism-weighted state count, defect density and lifetime, the mass-gap estimate obtained from minimum action differences, correlation lengths, spectral dimension, and the free energies of competing sectors. Gauge invariance is tested by generating random families of local automorphisms, applying them to representative configurations, and verifying that the action, the partition function, the defect class labels, and all gauge-invariant observables remain unchanged to machine precision. Scaling analyses are performed by systematically enlarging the complex, refining its triangulation, or increasing its topological complexity while holding boundary conditions fixed. Finite-size behavior, approach to continuum tendencies, stability of the mass gap, persistence of defects, and convergence of effective dimension are tracked as functions of system size. Any quantity that drifts or diverges under refinement is flagged for further analytic study.

12.3 Minimal and Strong Success Criteria

Minimal success requires that the model be defined rigorously, that gauge invariance of the action and observables be proved algebraically and verified numerically, that ordinary lattice gauge theory be recovered exactly in the one-object limit, and that the complete finite partition

function be computed for at least one nontrivial example. These requirements establish internal consistency and computational tractability. Strong success requires in addition at least one result that is not inserted by definition. Examples include the dynamical emergence of a preferred automorphism sector, the appearance of a quantized stable defect spectrum, a nonzero mass gap that remains stable under controlled refinement, a universal relation between defect charge and minimum coherence cost, or the convergence of effective dimension to a value independent of the underlying triangulation. Strong success demonstrates that the equivalence-first dynamics generates new structure rather than merely repackaging known mathematics.

12.4 Explicit Falsification Conditions

The framework is falsifiable at several levels. It would require substantial revision if no physically meaningful admissibility selector can be defined that distinguishes realizable identifications from arbitrary formal morphisms while preserving composition, inverses, and relevant observables. It would be rejected if the coherence action fails to be invariant under local changes of presentation, if the one-object reduction does not reproduce ordinary lattice gauge theory cleanly, or if the proposed mass, defined as minimum invariant coherence cost, depends on the choice of gauge representative, triangulation, or labeling. The finite quantum sum would be inconsistent if it diverges or becomes ill-defined within its intended domain of finite groupoids and finite complexes. Finally, the model would be regarded as empty if every claimed result merely restates an input assumption without additional constraint. These conditions are stated so that failure can be recognized unambiguously and so that any revision can be targeted at the specific layer that has failed.

12.5 Research Sequence and the Finite Kernel of a Candidate Theory of Everything

The recommended progression begins with the finite groupoid gauge dynamics already constructed. The next steps are the computation of defect spectra and mass gaps on successively larger complexes, the controlled introduction of higher-groupoid data, the derivation of a continuum gauge-field limit, the formulation of quantum measurement and probability from truncation maps, the emergence of geometry from relational distance and fiber entropy, the thermodynamic formulation of gravity, the dynamical selection of Standard-Model-like gauge structure, and finally phenomenology. At each stage the finite model supplies exact benchmarks against which approximations and extrapolations can be tested. The present construction does not claim to be a completed Theory of Everything. It is offered as the finite kernel from which a candidate theory would still have to derive the observed gauge group, fermionic and chiral matter, three generations, the Higgs sector, particle masses and mixing angles, Lorentzian three-plus-one-dimensional geometry, general relativity, quantum probabilities, and experimentally distinguishable predictions. A successful finite model would

establish that a single dynamics of admissible equivalence can generate these structures rather than merely accommodate them.

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